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Underwriting and Actuarial Consulting, Training and Research

Casualty Excess Pricing Using Power Curves

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CARe Seminar

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Agenda

- **Excess pricing approaches**
  - US vs. non-US
  - Property vs. casualty
- **Power curves**
  - Rationale and assumptions
  - The alpha parameter
  - General formulae
- **Practical issues and considerations**

Useful references

- Swiss Re publications on Pareto Rating
- Miccolis, Robert S. On the Theory of Increased Limits and Excess of Loss Pricing. *Proceedings of the Casualty Actuarial Society* 1977: LXIV 27-59
- Paul Riebesell. Einführung in die Sachversicherungsmathematik, Berlin 1936.
- Thomas Mack and Michael Fackler. Exposure Rating in Liability Reinsurance, ASTIN 2003.

Excess pricing:
US vs. Non-US Classes

USA Classes

- ISO curves for property and casualty classes
- Salzmann (1963) and Hartford (1991) curves for property
- Internally developed (“fitted”) loss models
- Brokers’ curves and models
- NCCI ELPPF for Workers Comp

Non-USA Classes

- Swiss Re first loss scales or exposure curves - property
- Power curves – casualty
- Internally developed loss models
- Minimum ROL
- Underwriters’ judgement

Excess pricing

- Insurance and reinsurance – same rationale
- Commonly used approaches
  - Loss distribution – ideal
  - Exposure curves
  - First loss scales
  - Increase limits factors
  - Excess factors
  - Rates per million of coverage
- Consider
  - Expense handling and risk loads
  - Currency and inflation adjustments

Exposure curves

- Assumptions:
  - Sum insured proxy for maximum possible loss
  - No incentive to under-insured
  - Direct relation between size of loss and amount at risk
  - Loss cost distribution by % of IV independent of sum insured
- Curves by peril or hazard
  - Fire
  - Wind
  - Hurricanes

Exposure curves as a loss elimination ratio

Limited expected value and loss elimination ratio

$$LEV_X(a) = E_X[X \wedge a] = \int_0^a f_X(x)dx + a(1 - F_X(a)) = \int_0^a (1 - F_X(x))dx$$

$$r_X(x) = \frac{E_X[X \wedge a]}{E_X(X)}$$

75% of losses are at 5% or less of Insured Value or Sum Insured

Loss as % of Insured Value	Sum Insured £100k	£200k	£300k	£500k
5%	75.0	74.6	76.5	72.5
10	78.0	77.5	78.1	78.6
20	81.0	80.3	81.9	80.5
30	84.0	83.2	83.5	84.2
40	89.0	87.5	88.6	88.9
50	91.0	90.8	91.2	91.3
60	93.0	94.5	92.7	92.7
70	95.0	95.2	95.6	95.8
80	98.0	97.8	98.2	98.9
90	99.0	98.7	99.4	99.5
100	100.0	100.0	100.0	100.0

Other considerations

- Consider all policy sections:
  - Building value
  - Contents
  - Other structures (e.g. attached garage)
  - Loss of use
- Thus, loss can be > 100% of sum insured
- Consider type of construction
- Curves are inflation independent
  - Loss increases in same proportion of sum insured

Exposure curves vs. ILFs

- Common (simplifying) assumption in property
  - Ratio $\frac{\text{Claim Size}}{\text{Sum Insured}}$ is constant for risks within group
- Assumptions does not hold in liability
  - Total limit purchased chosen by p/h
  - Limit ≠ maximum loss exposure
  - Dependence between limit and severity?
  - Dependence between limit purchased and exposure base (turnover, payroll, fee income, etc)?

Challenges when deriving ILFs

- Claim severity vs. limit purchased
- Claim severity vs. exposure base
- Assumption of independence between frequency and severity
- Aggregate vs. any one claim limits
- IBNER and claims reserving practices
- Lack of policy information at claim level
- Changes of mix of business year on year
- Multiple claim records for same underlying claim

The power curves

- Commonly used in London Market
  - Non-US liability classes
  - EL, GL, D&O, PI, FI, etc
- Insurance and reinsurance excess pricing
- Other names:
  - Alpha curves
  - Riebesell's curves
  - The German method
- Introduced by Riebesell in 1936

The rationale behind power curves

- Basic limit = B, pure premium P(B)
- Pure premium for limit 2B = P(B)*(1+r), e.g. r=20%
- Pure premium for limit 4B = P(B)*(1+r)(1+r)
- **Riebesell's rule:**

$$P(2^k B) = P(B) * (1 + r)^k$$

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The rationale behind power curves

- For any limit L, we can generalise

$$P(L) = P(B) * \left(\frac{L}{B}\right)^{ld(1+r)} = LEV_X(L)$$

where ld() is the binary log and 0 < z < 1

- ld(1+z) is usually called the alpha parameter

The rationale behind power curves

- The loss elimination ratio is:

$$r_X(x) = \frac{LEV_X(x)}{LEV_X(SI)} = \left(\frac{x}{SI}\right)^{ld(1+r)} < 1$$

- The ILF formula

$$ILF(L) = \frac{LEV_X(L)}{LEV_X(B)} = \left(\frac{L}{B}\right)^{ld(1+r)} > 1 \text{ if } L > B$$

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“Nice” properties

- LER only depends on ratio of x to SI
- Properties of exposure curves apply
- Scale invariant
  - Inflation
  - Currency
- Easy closed form formula
- Pareto tail

What is the distribution underlying the power curves?

- Mack and Fackler 2003: For any parameters of Riebesell’s rule we can construct a claims size distribution underlying such rule.
- The claim size distribution is a one parameter Pareto over a threshold u and parameter < 1.
- Below the threshold u?
  - the distribution is constant (independent of x)
- The threshold u could be a problem if deductibles or reinsurance attachment are low

Practical uses

- Underwriters are primary users of these curves
- Difficult to find an "alpha" that works ground up
  - Different alphas depending on original policy's excess or deductible
  - Common to ignore deductibles and attachments from exposure rating formula

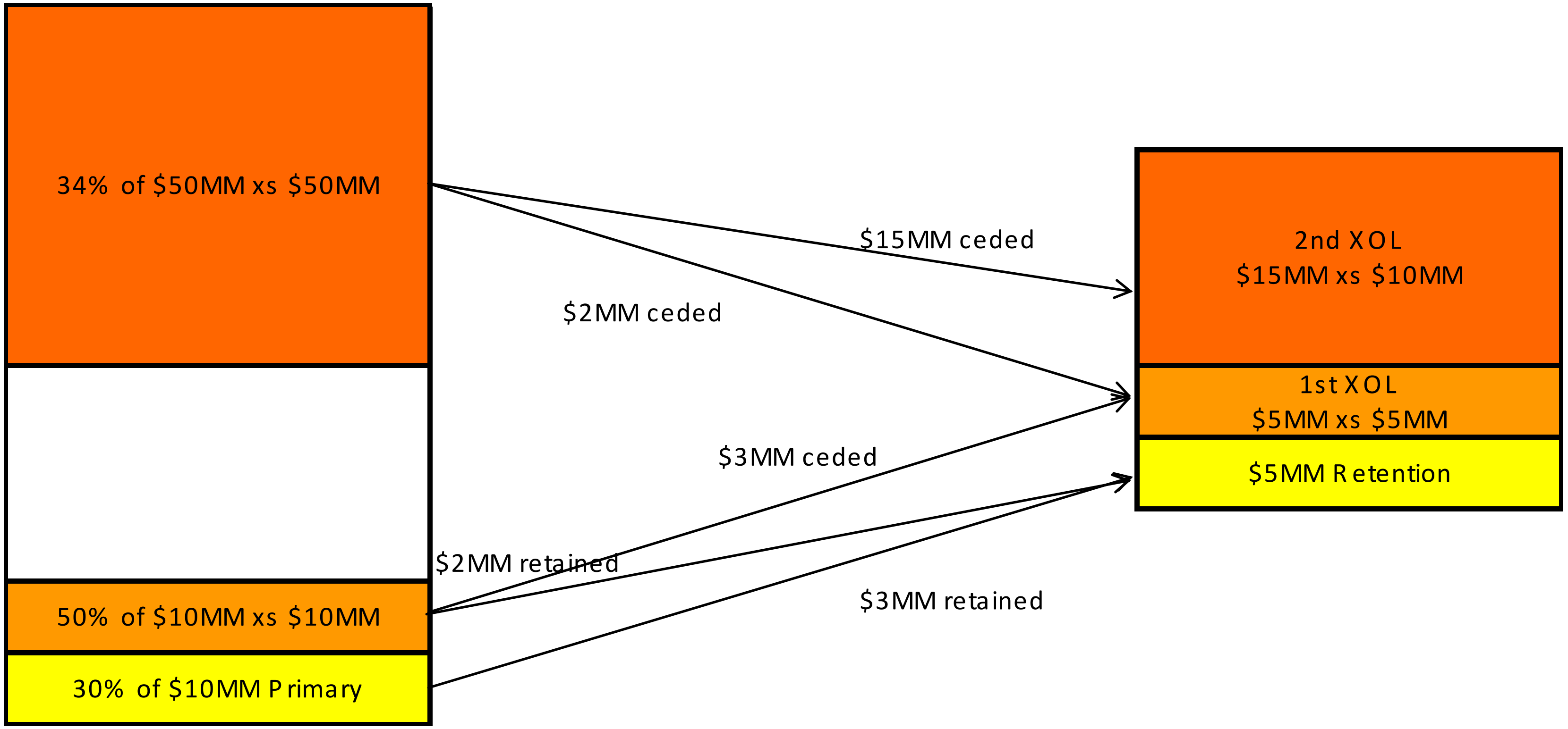
$$\%\,\text{loss in layer} = \frac{\text{ILF}(\text{TLimit} + \text{TXS} + \sout{\text{PXS} + \text{D}}) - \text{ILF}(\text{TXS} + \sout{\text{PXS}})}{\text{ILF}(\text{PLimit} + \sout{\text{PXS}}) - \sout{\text{ILF}(\text{PXS})}}$$

Practical issues

- Exposure rating with co-insurance – stacking of limits
  - Discontinuities
- Currency independent – OK
- Inflation independent – realistic?
  - Experience vs. exposure rating results may show significant differences
- ALAE or cost assumptions
  - Curves rarely adjusted for this

Power curves with stacking of limits

$25MM capacity spread over various layers



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Power curves with stacking of limits

- Exposure rate \$5MM P/O \$10MM xs \$10MM in the 1st RI excess: \$5MM xs \$5MM
- Assume r = 30%, alpha = 0.3785
- \$2MM are retained, \$3MM are ceded
- Ground up basis:
  - RI loss cost is 50% of \$6MM xs \$14MM P/O \$10MM xs £10MM

$$\% \text{ of loss in layer} = \frac{\text{ILF}(20) - \text{ILF}(14)}{\text{ILF}(20) - \text{ILF}(10)} = 54.73\%$$

- Ignoring original excess
  - RI loss cost is 50% of \$6MM xs \$4MM P/O \$10MM

$$\% \text{ of loss in layer} = \frac{\text{ILF}(10) - \text{ILF}(4)}{\text{ILF}(10)} = 29.31\%$$

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ALAE treatment and exposure rating

Apply following adjustments:

		ALAE in Curve	
Insurance Policy	**Reinsurance Treaty**	**Indemnity**	**Indemnity + ALAE**
Included	**Included**	Adjust reinsurance limit and attachment	OK
Included	**Pro-rata in addition**	Adjust policy limit	Adjust policy limit and reinsurance limit and attachment
In addition	**Included**	Adjust reinsurance limit and attachment	Adjust policy limits
In addition	**Pro-rata in addition**	OK	Adjust reinsurance limit and attachment

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Power curves:
final remarks

Advantages

- Underwriters are “comfortable”
- Aligned with UW mindset
- Common alphas across the market
- Scale invariant (ok for currency)
- Closed form formula (no interpolation)
- Pareto tail (?)

Disadvantages

- Difficult to replace
- Validity of assumptions
- Odd results:
  - High limits and excess on excess
  - Share of limits and stacking of policies
  - Low original deductibles
- Inflation independent
- ALAE/costs assumptions